SAMPLE PAPER

CLASS - XIIth GSEB (ENGLISH MEDIUM)

Test Type : FULL SYLLABUS Test Pattern : BOARD PATTERN

MATHEMATICS

Time Allowed : 3 Hours **Maximum Marks : 100**

- Please check that this question paper contain **09** printed pages.
- Please check that this question paper contains **68** questions.
- Please write down the serial number of the question before attempting it.

IMPORTANT INSTRUCTIONS

1. All questions are compulsory.

2. There are 2 Sections A and B respectively.

SECTION-A (Objective Questions)

Questions No. 1 to 50 carry 1 Marks each = 50 Marks (Only one correct)

SECTION-B (Subjective Questions)

Questions No. 51 to 58 carry 2 Marks each = 16 Marks

Questions No. 59 to 64 carry 3 Marks each = 18 Marks

Questions No. 65 to 68 carry 4 Marks each = 16 Marks

You may use the following values of physical constants wherever necessary :
$c = 3 \times 10^8\ ms^{-1}$, $h = 6.63 \times 10^{-34}$ Js, $e = 1.6 \times 10^{-19}$ C, $\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1},

$\varepsilon_0 = 8.854 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$, $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ N\ m^2\ C^{-2}$, $m_e = 9.1 \times 10^{-31}$ kg

Mass of neutron 1.675×10^{-27} kg, Mass of proton 1.673×10^{-27} kg
Avogadro's number = 6.023×10^{23} per gram mole
Boltzmann's constant $1.38 \times 10^{-23}\ JK^{-1}$

MATHEMATICS

PART - A

* **Choose correct answer from the given alternatives and write it. [Each carries 1 Mark]** **[50]**

1. A = {1, 2, 3} the number of equivalence relations containing (1, 3) is ....

(1) 1 (2) 2 (3) 3 (4) 8

2. If $f : [0, \pi] \rightarrow [-1, 1]$ is one-one and onto, then ..... is possible.

(1) $f(x) = |x|$ (2) $f(x) = \sin x$ (3) $f(x) = x^3$ (4) $f(x) = \cos x$

3. If $a * b = \frac{ab}{100}$ on Q^+, inverse of 0.1 is .....

(1) 100000 (2) 10000 (3) 1000 (4) 10

4. If $f : R \rightarrow R$, $g : R \rightarrow R$, $f(x) = 3x - 2$ and $g(x) = x^2 + 1$ then $(gof^{-1})(2) =$ .... .

(1) $\frac{25}{9}$ (2) $\frac{25}{3}$ (3) $\frac{16}{9}$ (4) $\frac{4}{3}$

5. If $\sin(2\tan^{-1} x) = 1$, then x = ...

(1) $\frac{1}{2}$ (2) $\frac{1}{\sqrt{2}}$ (3) 1 (4) $\sqrt{3}$

6. If $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$, then $\cos^{-1} x + \cos^{-1} y =$ ..... .

(1) $\frac{5\pi}{3}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{6}$ (4) π

7. If $\sin^{-1}\left(\frac{x}{13}\right) + \sec^{-1}\left(\frac{13}{5}\right) = \frac{\pi}{2}$, then the value of x is ....

(1) 1 (2) 5 (3) 12 (4) 13

8. The value of $\tan\left(\frac{1}{2}\cos^{-1}\frac{3\sqrt{5}}{7}\right)$ is ..... .

(1) $-\frac{\sqrt{5}+1}{4}$ (2) $\frac{7-3\sqrt{5}}{2}$ (3) $\frac{7+3\sqrt{5}}{2}$ (4) $\frac{\sqrt{5}-1}{4}$

9. If $D = \begin{vmatrix} 2 & \cos\theta & 2 \\ -\cos\theta & 2 & \cos\theta \\ -2 & -\cos\theta & 2 \end{vmatrix}$, then value of D lies in the interval ....

(1) $(16, \infty)$ (2) $(16, 20)$ (3) $[12, 16]$ (4) $[16, 20]$

10. If l, m, n are real numbers such that $l^2 + m^2 + n^2 = 0$ then $\begin{vmatrix} 1+l^2 & lm & ln \\ lm & 1+m^2 & mn \\ nl & mn & 1+n^2 \end{vmatrix} =$ ....

(1) 0 (2) 1 (3) $l + m + n + 2$ (4) $lmn - 1$

11. If $f(\alpha)=\begin{vmatrix}1 & \alpha & \alpha^2\\ \alpha & \alpha^2 & 1\\ \alpha^2 & 1 & \alpha\end{vmatrix}$, then $f(\sqrt[3]{3})=$ ....

(1) –4 (2) 4 (3) 2 (4) –2

12. A is 2 × 3 matrix, if A^TB and BA^T are defined, then B is a .... matrix.

(1) 2 × 3 (2) 3 × 2 (3) 2 × 2 (4) 3 × 3

13. If $\begin{bmatrix}2 & x & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 1 & 0\end{bmatrix}\begin{bmatrix}2\\ 1\\ x\end{bmatrix}=0$, then x = ...

(1) –3 (2) 3 (3) 6 (4) –6

14. If A (adj A) = 4I, where A is 3 × 3 matrix, then |A| = ...

(1) 1 (2) 2 (3) 4 (4) 8

15. If $A=\begin{bmatrix}\alpha^2 & 5\\ 5 & -\alpha\end{bmatrix}$ and $|A^{10}|=1024$, then α = ...

(1) 2 (2) –2 (3) 3 (4) –3

16. $\dfrac{d^2x}{dy^2}=$.....

(1) $\dfrac{1}{\frac{d^2y}{dx^2}}$ (2) $\dfrac{1}{\left(\frac{dy}{dx}\right)^2}$ (3) $-\dfrac{1}{\left(\frac{dy}{dx}\right)^2}$ (4) $-\dfrac{1}{\left(\frac{dy}{dx}\right)^3}.\dfrac{d^2y}{dx^2}$

17. If $f(x) = \log_3(\log_5 x)$, then f'(x) = ...

(1) $\dfrac{1}{x\log_e x\log 3}$ (2) $\dfrac{1}{x\log_e x}$ (3) $\dfrac{1}{x\log_e 3\log_e 5}$ (4) $\dfrac{1}{x\log_e x\log_e 5}$

18. If we apply the Rolle's theorem to $f(x) = e^x \cos x$, $x\in\left[\dfrac{\pi}{2},\dfrac{3\pi}{2}\right]$, then c = ...

(1) $\dfrac{3\pi}{4}$ (2) $\dfrac{5\pi}{4}$ (3) π (4) $\dfrac{15\pi}{4}$

19. $\displaystyle\int\frac{3\tan\frac{x}{3}-\tan^3\frac{x}{3}}{1-3\tan^2\frac{x}{3}}dx=.....+c$

(1) log|tan x| (2) – log|cos x| (3) $\sec^2 x$ (4) –log |sec x|

20. $\int \frac{dx}{x^2(x^4+1)^{\frac{3}{4}}} = ... + c$

(1) $\left(1+\frac{1}{x^4}\right)^{\frac{1}{4}}$ (2) $-\left(1+\frac{1}{x^4}\right)^{\frac{1}{4}}$ (3) $-\frac{1}{4}\left(1+\frac{1}{x^4}\right)^{\frac{1}{4}}$ (4) None of these

21. $\int \cos^{-\frac{3}{7}} x \sin^{-\frac{11}{7}} x dx = ...$.

(1) $\log\left|\sin^{\frac{4}{7}} x\right| + c$ (2) $\frac{4}{7}\tan^{\frac{4}{7}} x + c$ (3) $-\frac{7}{4}\tan^{-\frac{4}{7}} x + c$ (4) $\log\left|\cos^{\frac{3}{7}} x\right| + c$

22. A problem in mathematics is given to three students whose probability of solving it are $\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$. The probability that at least one of them solves the problem is ....

(1) $\frac{1}{27}$ (2) $\frac{19}{27}$ (3) $\frac{8}{27}$ (4) $\frac{26}{27}$

23. If A and B are two independent events such that $P(A) = \frac{1}{2}, P(B) = \frac{1}{5}$, then $P(A \mid A \cup B) = ...$.

(1) $\frac{1}{6}$ (2) $\frac{1}{2}$ (3) $\frac{1}{10}$ (4) $\frac{5}{6}$

24. A fair die is rolled 6 times. If "getting an even number" is a success, then probability of 5 successes is

(1) $\frac{5}{64}$ (2) $\frac{3}{32}$ (3) $\frac{63}{64}$ (4) $\frac{5}{6}$

25. If the probability of non-defective screw is 0.9, the mean and standard deviation for the binomial distribution of defective screws in a total of 500 screws are respectively ....

(1) 50, 6·71 (2) 500, 6·71 (3) 50, 45 (4) 50, 7·71

26. The probability of India winning a one day match against West Indies is $\frac{1}{2}$. The probability that in a 5 match series Indies India's Second win occurs at the third one day is ..... .

(1) $\frac{1}{8}$ (2) $\frac{1}{4}$ (3) $\frac{1}{2}$ (4) $\frac{2}{3}$

27. Let x and y be optimal solution of a linear programming problem, then ....

(1) $z = \lambda x + (1 - \lambda)y, \lambda \in R$ is also an optimal solution

(2) $z = \lambda x + (1 - \lambda)y, 0 \le \lambda \le 1$ gives an optimal solution

(3) $z = \lambda x + (1 + \lambda)y, 0 \le \lambda \le 1$ gives an optimal solution

(4) $z = \lambda x + (1 + \lambda)y, \lambda \in R$ gives an optimal solution

28. Solution of the following linear programming problem : Minimize $z = -3x + 2y$ subject to $0 \le x \le 4$, $1 \le y \le 6$, $x + y \le 5$ is ....

(1) –10 (2) 0 (3) 2 (4) 10

29. $f(x) = (x + 2)e^{-x}$ is increasing in .... $x \in R$

(1) $(-\infty, -1)$ (2) $(-1, -\infty)$ (3) $(2, \infty)$ (4) R^+

30. The rate of change of volume of a cylinder w.r.t. radius whose radius is equal to its height is

(1) 4 (area of base) (2) 3 (area of base) (3) 2 (area of base) (4) (are of base)

31. The normal to $x^2 = 4y$ passing through (1, 2) has equation ......

(1) $2x = y$ (2) $x + y - 3 = 0$ (3) $2x + 3y - 8 = 0$ (4) $x - y + 1 = 0$

32. The local minimum value of $x^2 + \frac{16}{x}$ is ..... $x \in R - \{0\}$

(1) 12 (2) 22 (3) –12 (4) 2

33. For what values of k, $f(x) = x^2 - kx + 20$ is strictly increasing on [0, 3]

(1) $k < 0$ (2) $0 < k < 1$ (3) $1 < k < 2$ (4) $2 < k < 3$

34. $\int x^5 e^{x^2} dx = \frac{e^{x^2}}{2} f(x) + c$, then f(x) = ...

(1) $x^4 - 2x^2 + 2$ (2) $x^4 + 2x^2 + 2$ (3) $x^4 - 2x^2 - 2$ (4) $x^4 + 2x^2 - 2$

35. If $I_n = \int (\log x)^n dx$ then $I_n + n\, I_{n-1}$ = ...

(1) $x(\log x)^n$ (2) $(x \log x)^n$ (3) $(\log x)^{n-1}$ (4) $n(\log x)^n$

36. $\int \left[\log\{\log(\log x)\} + \frac{1}{\log x \cdot \log(\log x)} \right] dx = ... + c$

(1) $x \log\{\log(\log x)\}$ (2) $e^x \log\{\log(\log x)\}$ (3) $x \log(\log x)$ (4) $x \log x$

37. $\int a^x [f'(x) + f(x)\log a] dx = ... + c.$ $(a \in R^+ - \{1\})$

(1) $a^x \cdot \log a\, f'(x)$ (2) $a^x \cdot f'(x)$ (3) $a^x \cdot f(x)$ (4) $a^x \cdot \log a \cdot f(x)$

38. $\int_0^9 \left[\sqrt{x} + 2 \right] dx = ...$ [.] represents G.I.F.

(1) 31 (2) 32 (3) 23 (4) 18

39. $\int_0^{4014} \frac{2^x}{2^x + 2^{4014-x}} dx = ...$

(1) 2007 (2) 4014 (3) 2^{4014} (4) 2^{2007}

40. $\int_0^{\pi} e^{\sin^2 x} \cos^3 x\, dx = ...$

(1) –1 (2) 0 (3) 1 (4) π

41. Area of the region bounded by the circle $x^2 + y^2 = 4$ and lines x = 0, x = 2 in the first quadrant is

(1) π (2) $\frac{\pi}{2}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{4}$

42. Find area of the region bounded by the parabola $y^2 = 4ax$ and its latus rectum x = a

(1) $\frac{8}{3}a$ (2) $\frac{4}{3}a^2$ (3) $\frac{8}{3}a^2$ (4) 2

43. An integrating factor of differential equation $\frac{dy}{dx} = \frac{1}{x+y+2}$ is ..... .

(1) e^x (2) e^{x+y+2} (3) e^{-y} (4) $\log|x + y + 2|$

44. The length of subnormal to the hyperbola $x^2 - y^2 = 8$ at the point (3, 1) is ...

(1) 3 (2) $\frac{1}{3}$ (3) 8 (4) $\frac{1}{8}$

45. The general solution of $(x + 2y^3)\frac{dy}{dx} - y = 0$ is

(1) $x = y^3 + Ay$ (2) $y(1 - xy) = Ax$ (3) $x(1 - xy) = Ay$ (4) $x(1 + xy) = Ay$

46. If position vectors of vertices of ΔABC are $\bar{a}, \bar{b}$ and $\bar{c}$ then vector perpendicular to its plane is

(1) $\bar{a}\times\bar{b}+\bar{b}\times\bar{c}+\bar{c}\times\bar{a}$ (2) $\frac{\bar{a}\times\bar{b}+\bar{b}\times\bar{c}+\bar{c}\times\bar{a}}{|\bar{a}\times\bar{b}+\bar{b}\times\bar{c}+\bar{c}\times\bar{a}|}$ (3) $\frac{\bar{a}\times\bar{b}}{|\bar{a}\times\bar{b}|}$ (4) None of these

47. If for ΔABC, $\overrightarrow{AB} = 3\hat{i} + 4\hat{k}$ and $\overrightarrow{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ then length of the medium drawn from A is :

(1) $\sqrt{288}$ (2) $\sqrt{18}$ (3) $\sqrt{72}$ (4) $\sqrt{33}$

48. If $\bar{x} = (1,2,4), \bar{y} = (-1, -2, k)$, $k \neq -4$ then $|\bar{x}\cdot\bar{y}| \ldots |\bar{x}||\bar{y}|$

(1) < (2) > (3) = (4) ≥

49. Line passing through (2, –3, 1) and (3, –4, –5) intersects ZX - plane at ....

(1) (–1, 0, 13) (2) (–1, 0, 19) (3) $\left(\frac{13}{6}, 0, \frac{-19}{6}\right)$ (4) (0, –1, 13)

50. Equation of the line L passing through A(–2, 2, 3) and perpendicular to $\overline{AB}$ is ..... where coordinates of B(13, –3, 13).

(1) $\frac{x-2}{3} = \frac{y+2}{13} = \frac{z+3}{2}$ (2) $\frac{x+2}{3} = \frac{y-2}{13} = \frac{z-3}{2}$

(3) $\frac{x+2}{15} = \frac{y-2}{-5} = \frac{z-3}{10}$ (4) $\frac{x-2}{15} = \frac{y+2}{-5} = \frac{z+3}{10}$

PART-B

Section - a

*** Answer the following Questions. [Each carries 2 marks] [16]**

1. If f: R $\rightarrow$ (–1,1), f(x) = $\frac{10^x - 10^{-x}}{10^x + 10^{-x}}$ and f is one-one and onto then find $f^{-1}(x)$.

2. Minimize z = 200x + 500 y subject to x + 2y $\geq$ 10, 3x+ 4y $\leq$ 24 and x $\geq$ 0, y $\geq$ 0

3. Find $\int \frac{x^2 e^x}{(x+2)^2} dx$

4. Find the general solution of differential equation $(1+y^2)\, dx = (\tan^{-1}y - x)\, dy$.

5. If $\bar{a}, \bar{b}$ and $\bar{c}$ are non-zero vectors and $\bar{a} \times \bar{b} = \bar{c}$ and $\bar{b} \times \bar{c} = \bar{a}$ then prove that $|\bar{b}| = 1$

6. The direction cosines l, m, n, of two lines satisfy l + m + n = 0 and $l^2 - m^2 + n^2 = 0$.
Find the measure of the angle between the lines.

7. Determine the interval in which following function is strictly increasing or strictly decreasing.
f: R $\rightarrow$ R $f(x) = 2x^3 - 12x^2 + 18x + 15$

8. Prove that $\begin{vmatrix} 1+x & 1 & 1 \\ 1+y & 1+2y & 1 \\ 1+z & 1+z & 1+3z \end{vmatrix} = 2xyz\left(3 + \frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right)$

Section - b

*** Answer the following Questions. [Each carries 3 marks] [18]**

9. Prove that the right circular cone of maximum volume inscribed in a sphere of radius r has altitude $\frac{4r}{3}$.

10. Find the area of region bounded by the circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 4x$.

11. Find the image (1,5,1) in the plane x – 2y + z + 5 = 0

12. Prove that $\tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right) + \tan\left(\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right) = \frac{2b}{a}$

13. A restaurant serves two special dises A and B to its customers consisting of 60% men and 40% women. 80% of men order dish A and the rest order B. 70% of women order B and the rest order A. In what ratio of dishes A to B should the restaurant prepare the two dishes?

14. By using reduced row echelon method, find the inverse of the matrix A= $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

Section - c

*** Answer the following Questions. [Each carries 4 marks] [16]**

15. The temperature of a body in a room is 100°F. After five minutes, the temperature of the body becomes 50°F. After another 5 minutes, the temperature becomes 40°F. What is the temperature of surroudings?

16. Find $\int_{0}^{1} \tan^{-1}\left(\frac{1}{1-x+x^2}\right)dx$

17. Find $\int \frac{1}{x^4+1}dx$

18. If $y = x \log\left(\frac{x}{a+bx}\right)$ then prove that $x^3y_2 = (xy_1-y)^2$